MATH 117 | Test 2, Solutions to the Individual portion
Dr. Fenton | Fall 2007

1. (10 pts.) Suppose $g(x) = \dfrac{2x^2+x+1}{9-x^2}$. Does g have a horizontal asymptote? If so, where? If not, why not?
ANSWER: To decide this, take the limit as $x \to \infty$.

$$\lim_{x\to\infty} \frac{2x^2+x+1}{9-x^2} = \lim_{x\to\infty} \frac{2+\frac{1}{x}+\frac{1}{x^2}}{\frac{9}{x^2}-1} = \frac{2}{-1} = -2$$

So there is a horizontal asymptote at $y=-2$. The limit as $x \to -\infty$ gives the same asymptote.

2. (5 pts.) Give an example of a function that is continuous at $x=0$ but is not differentiable there.
ANSWER: There are many possible examples. One easy example is the absolute value function $y = |x|$. The limit as $x \to 0$ is 0 and this equals the output $|0| = 0$.

3. (10 pts.) For the implicit function defined by $\cos(x+y) = xy+5$, find y' at the point $(2,-2)$.
ANSWER: First differentiate the entire equation to get

$$-\sin(x+y)\cdot(1+y') = x\cdot y' + y\cdot 1 + 0$$

At the point $(2,-2)$ this becomes $-\sin(2-2)\cdot(1+y') = 2\cdot y' + (-2)\cdot 1 + 0$, so $y'=1$.

4. (45 pts.) Find the derivative of each function listed here. Simplify your answers. (5 pts. each)
   (a) $f_1(x) = \tan(4x)$ ANSWER: $4\sec^2(4x)$
   (b) $f_2(x) = (4-x^2)^{100}$ ANSWER: $-200x(4-x^2)^{99}$
   (c) $f_3(x) = \dfrac{x-2}{x-1}$ ANSWER: $\dfrac{1}{(x-1)^2}$
   (d) $f_4(x) = 5x^7 - 7x^5 + 4x^4 - 13x^2 + 54x - 106$ ANSWER: $35x^6 - 35x^4 + 16x^3 - 26x + 54$
   (e) $f_5(x) = \tan^{-1}(4x)$ ANSWER: $\dfrac{4}{1+16x^2}$
   (f) $f_6(x) = e^{\cos(x)}$ ANSWER: $e^{\cos(x)}\cdot(-\sin(x))$
   (g) $f_7(x) = \dfrac{\sin(x)}{x^2}$ ANSWER: $\dfrac{x\cos(x) - 2\sin(x)}{x^3}$
   (h) $f_8(x) = \sqrt{x^2-x}$ ANSWER: $\dfrac{2x-1}{2\sqrt{x^2-x}}$
   (i) $f_9(x) = xe^x$ ANSWER: $xe^x + e^x = e^x(x+1)$

5. (5 pts.) Here is a graph of a function $F(x)$. Sketch (neatly, please!) a graph of $F'(x)$.
ANSWER: Sorry, I haven't figured out how to include graphics in these files. Going left to right, the graph should rise along the horizontal axis in quadrant II, then decrease through the origin into quadrant IV, then increase asymptotically along the horizontal axis. The derivative should be an odd function.

MATH 117 — Test 2, Group portion

Dr. Fenton — Fall 2007

1. (7 pts.) Is there a value of c that makes the following function continuous at $x = 0$? Explain why or why not.

$$f(x) = \begin{cases} \frac{\sin(10x)}{2x} & \text{if } x < 0 \\ c & \text{if } x = 0 \\ \sqrt{5}\cdot\sqrt{3x+5} & \text{if } x > 0 \end{cases}$$

ANSWER: Yes, $c = 5$ makes this continuous. The limit of f as $x \to 0$ must equal the output value. It is necessary to check both sides for the limit, $x \to 0^-$ and $x \to 0^+$. However, both limits equal 5.

2. (9 pts.) Let $g(x) = \dfrac{x-2}{x-1}$. Find an equation for the tangent line at $x = 3$. Then create a graph showing both g and this tangent line.

ANSWER: The derivative of this function is $\dfrac{1}{(x-1)^2}$. At $x = 3$ the y-coordinate is $\frac{1}{2}$ and the slope is $\frac{1}{4}$, so the tangent line is $y - \frac{1}{2} = \frac{1}{4}(x-3)$. To plot the function and the tangent line, the Maple command is

$$plot(\{\frac{1}{(x-1)^2}, \frac{1}{4}x - \frac{1}{4}\}, x=0..5)$$

(Your domain may be different, but should include 3.)

3. (9 pts.) Our textbook gives two versions of the definition of derivative:

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

and

$$f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$$

Use the function $f(x) = (x^2+1)^{-1}$.

(a) With this function f, calculate $\dfrac{f(a+h)-f(a)}{h}$ and simplify the result.

ANSWER: $\dfrac{\frac{1}{(a+h)^2+1} - \frac{1}{a^2+1}}{h} = \dfrac{-(2a+h)}{(a^2+2ah+h^2+1)(a^2+1)}$

(b) Take the limit of your answer to part (a) to find $f'(a)$.

ANSWER: $\dfrac{-2a}{(a^2+1)^2}$

(c) With the same function f, calculate $\dfrac{f(x)-f(a)}{x-a}$ and simplify the result.

ANSWER: $\dfrac{\frac{1}{x^2+1} - \frac{1}{a^2+1}}{x-a} = \dfrac{-(a+x)}{(x^2+1)(a^2+1)}$

(d) Take the limit of your answer to part (c) to find $f'(a)$.

ANSWER: This is the same as part (b), namely $\dfrac{-2a}{(a^2+1)^2}$.