MATH 117, Fall 2007 — LAB ACTIVITIES, Set 4

Sections 3.1, 3.2, 3.3, 3.4, 3.5 — due Friday, October 5 by 5:00

Objectives:

- to learn basic computational rules for derivatives (Section 3.1, 3.2, 3.3)
- to discover a value for the base of the natural exponential function (Section 3.1)
- to investigate certain limits involving trigonometric functions (Section 3.3)
- to understand the derivative of a composition (Section 3.4)
- to understand implicitly-defined functions (Section 3.5)
- to calculate slope for an implicitly-defined function (Section 3.5)

Activities

1. The Maple command *diff(f(x), x)* will calculate the derivative of whatever function $f(x)$ you type into the command. Use this to calculate the derivatives of the following functions. (You may wish to use *simplify* occasionally.)
   (a) $x^7, \dfrac{1}{x^3}, \sqrt[5]{x^3}, \dfrac{1}{\sqrt[3]{x^2}}$
   (b) $2x^5, 10x^{-2}, -3t^{\frac{4}{3}}$
   (c) $\sin x, \cos t, \tan y, \sec x, \csc v, \cot z$
   (d) $2^x, 5^x, e^x$
   (e) $2y^3 - y + 1, x^4 - 4x^3 + 6x^2 - 4x + 1$
   (f) $x \sin x, x^3 \cos x$
   (g) $\dfrac{\sin x}{x^2}$
   (h) $\dfrac{1+t}{\sqrt{1+t^2}}$

2. Look again at your results from Activity 1. Make up rules for each of the following situations
   (a) the derivative of a power function, x^a
   (b) the derivative of a trigonometric function
   (c) the derivative of an exponential function, a^x
   (d) the derivative of a sum of two functions, $f(x) + g(x)$
   (e) the derivative of a constant multiple of a function, $c \cdot f(x)$
   (f) the derivative of a product of two functions, $f(x) \cdot g(x)$
   (g) the derivative of a quotient of two functions, $\dfrac{f(x)}{g(x)}$

3. When we use the definition to calculate the derivative of an exponential function a^x, we get the following:
$$\frac{d}{dx}a^x = \lim_{h\to 0} \frac{a^{x+h} - a^x}{h} = a^x \cdot \lim_{h\to 0} \frac{a^h - 1}{h}$$
   Experiment with different values for a until $\displaystyle\lim_{h\to 0} \frac{a^h - 1.0}{h}$ equals 1. Get an answer accurate to at least five decimal places.

4. Use Maple to evaluate the following limits.

   (a) $\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta}$

   (b) $\lim_{\theta \to 0} \frac{\sin(5\theta)}{\theta}$

   (c) $\lim_{\theta \to 0} \frac{\sin(4\theta)}{9\theta}$

   (d) $\lim_{\theta \to 0} \frac{\cos(\theta) - 1}{\theta}$

   (e) $\lim_{\theta \to 0} \frac{\cos(3\theta) - 1}{4\theta}$

   Then give a general formula for $\lim_{h \to 0} \frac{\sin(a\theta)}{b\theta}$ and for $\lim_{h \to 0} \frac{\cos(a\theta) - 1}{b\theta}$.

5. Use Maple to calculate the derivatives of the following functions.

   (a) $\sin(3x), \cos(x^2), (7x - 4)^{-5}$

   (b) $\sqrt{1 - \sin x}$

   (c) $e^{\sin(1-u^2)}$

   (d) $\sec(\sqrt{9 + v^2})$

   Now give a general formula for the derivative of a composition, $(f \circ g)(x)$.

6. The equation $x^3 + y^3 = 9xy$ does not describe a function in the usual way. However, there is a function (several, in fact) implied by this equation, and we can calculate the derivative of this *implicit function*.

   (a) Use the Maple commands

   with(plots):

   implicitplot($x^3 + y^3 = 9xy$*, x=??..??, y=??..??)*

   to see a graph of this equation. (You must fill in values for the plotting ranges.)
   Maple hint: To improve the quality of this graph, you can increase the number of points by including *numpoints = ??* in the plotting command. Try 3000 points or more.

   (b) Then use the command

   implicitdiff($x^3 + y^3 = 9xy$*,y,x)*

   to calculate the derivative.

   (c) Calculate the slope of the tangent line at the point $(4, 2)$. On your graph, draw the tangent line at this point.

   (d) Explain what is happening to the derivative at the origin.