MATH 205 FG Test 2 Solutions
Dr. Fenton Fall 2008

1. (15 pts.) According to the American Red Cross, 10% of their donors have Type B blood. Suppose we examine a random sample of thirty donors and count how many of them have Type B blood.
   (a) What kind of distribution fits this situation, and why?
   This is a binomial distribution, specifically $B(30, 0, 10)$. Each trial (donor) gives a Yes/No answer, the donors are independent, and the probability of Type B blood stays the same.
   (b) Find the mean and standard deviation of this distribution.
   The mean is $30 \cdot 0.10 = 3$. The standard deviation is $\sqrt{30 \cdot 0.10 \cdot 0.90} = 1.64$.
   (c) Calculate $Pr(X \leq 5)$. Explain how you do this.
   With the **binomcdf** command, do binomcdf(30,0.1,5) = 0.9268.

2. (10 pts.) The mean monthly rent for a random sample of 10 apartments advertised in the local newspaper is \$635. Assume that the standard deviation is $\sigma = \$80$. How large a sample would be needed to estimate the mean rent μ to within $\pm\$25$ with 95% confidence?
   $n = \left(\frac{1.96 \cdot 80}{25}\right)^2 = 39.34$ which we round up to 40. (The precise z^* value for 95% confidence is 1.96.)

3. (10 pts.) In the year 2000, the National Assessment of Educational Progress test was given to high school seniors across the United States. The scores on this test were approximately normally distributed with the mean $\mu = 300$ and standard deviation $\sigma = 35$.
   (a) Suppose we pick one of these high school seniors at random. What is the probability that this student had a score higher than 320?
   The calculation is normalcdf(320,∞,300,35). I used 1000 for the upper limit (instead of ∞) and got 0.284 for the probability.
   (b) Now suppose we pick a random sample of eight of these high school seniors. What is the probability that the mean score for this sample of students is higher than 320?
   The calculation is the same except that the standard deviation must be divided by $\sqrt{8}$. This gives normalcdf(320,∞,300,$\frac{35}{\sqrt{8}}$) = 0.053.

4. (20 pts.) The Coca-Cola Company says that a 12-ounce can of Diet Coke contains 45.60 mg of caffeine. To find out if this is correct, a consumer group examines a random sample of 120 cans of Diet Coke and finds a mean of 45.53 mg of caffeine.
   (a) State the null and alternative hypotheses for this situation. Label which is which.
   The null hypothesis is $H_0 : \mu = 45.60$. The alternative hypothesis is $H_a : \mu \neq 45.60$.
   (b) Explain what Type I and Type II errors mean in this particular situation. (A general answer is not enough. Tell what they mean for these hypotheses.)
   A Type I error is rejecting the null hypothesis when it is true. For this situation, it means that we do not believe that 45.60 is correct, even though it is. A Type II error means accepting the null hypothesis when it is false. For this situation, it menas that we believe that 45.60 is correct when it is not.
   (c) We do not know what the distribution of caffeine levels is for the population of all cans of Diet Coke. Why are we able to use the normal distribution to test these hypotheses?
   Because we are using a sample—and a fairly large sample—the Central Limit Theorem says that the sampling distribution will be approximately normal.
   (d) Using a standard deviation of $\sigma = 0.60$ mg, find the P-value for the hypothesis test. Then decide which hypothesis you agree with and explain why.
   Use the **Z-Test** command with the values $\mu_0 = 45.60$, $\sigma = 0.60$, $\overline{x} = 45.53$, $n = 120$, and the alternative hypothesis $\mu \neq \mu_0$. This gives $P = 0.20$. This is not very small, so the evidence

against H_0 is not strong. We should accept 45.60 mg as the mean.

5. (15 pts.) Internet sites often vanish or move, so that references to them cannot be followed. In fact, 13% of Internet sites cited in major scientific journals are lost within two years after publication.
   (a) If a scientific paper contains seven Internet references, what is the probability that all seven are still good two years later?
   The probability that any one reference is still good is $1 - 0.13 = 0.87$. For seven references, which we must assume are independent, the probability that all seven are still good is $0.87^7 = 0.377$.
   (b) If a scientific paper contains seven Internet references, what is the probability that none of them are still good two years later?
   The probability that none of the seven are still good is the same as the probability that all seven are lost. This is $0.13^7 = 0.0000006$ (very tiny!).

6. (10 pts.) When working with confidence intervals, what are two ways to shrink the margin of error?
One way is to reduce the confidence level. This makes z^* smaller and makes the margin of error smaller. The other way to increase the sample size, that is, collect more data. This increases the value of n, which decreases the margin of error.

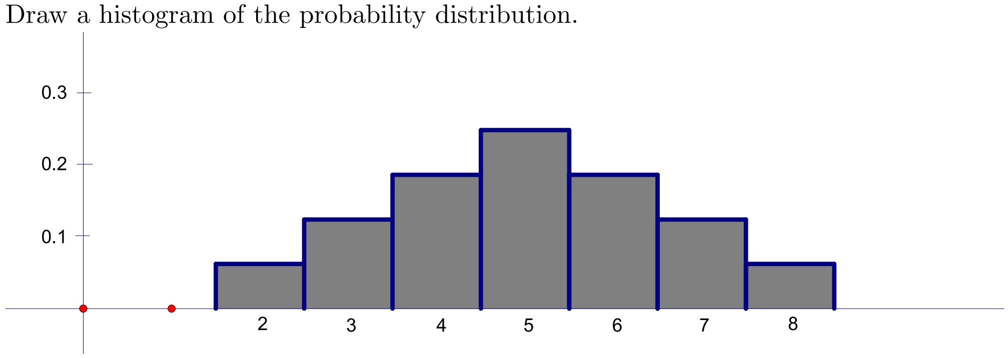
7. (20 pts.) Suppose that you have two pyramid-shaped dice, like the ones we used in class. Each of these dice is numbered from 1 to 4. The experiment is to roll the dice and record the sum of the two values.
   (a) List all possible pairs of values from rolling the two dice.

1,1	1,2	1,3	1,4
2,1	2,2	2,3	2,4
3,1	3,2	3,3	3,4
4,1	4,2	4,3	4,4

   (b) Calculate the probability of each possible sum.

Sum	2	3	4	5	6	7	8
Prob.	0.0625	0.125	0.1875	0.25	0.1875	0.125	0.0625

   (c) Draw a histogram of the probability distribution.



   (d) Calculate the mean μ and the standard deviation σ for this distribution.
   Put the sums $2, 3, \ldots, 8$ into the list **L1**. Then put the probabilities into **L2**. Using **1-Var Stats L1, L2** gives $\mu = 5$ and $\sigma = 1.58$.

BONUS QUESTION (5 pts.) Part of the scoring for this test is that everyone also will get half of the mean of their group's individual scores. When I have done this in other classes, the means of the groups almost always have been close to the same value. Explain why this happens.

This is the Central Limit Theorem again. Because I am calculating the means of the groups, I am using a sampling distribution. Sample means have less variability than individual scores because the standard deviation is smaller.