

**Appendix: Stuck Behind the Math: Just How Helpful Can One Expect Technology to be in Economics Pedagogy?**

**Frank Raymond<sup>1</sup>**  
**Anne Raymond<sup>2</sup>**  
**Myra McCrickard<sup>3</sup>**

In this appendix we provide eight examples from intermediate microeconomic and macroeconomic theory. The first example illustrates utility functions and indifference curves. The second illustrates consumer maximization. The third represents the producer problem of cost minimization. The last example demonstrates how *MAPLE* can be used to solve a Keynesian macroeconomic system.

While students may grasp the concept of an indifference curve, they often have difficulty envisioning a single indifference curve as part of an indifference map. This first example can be used to familiarize the student with the basic syntax of *MAPLE* while depicting the indifference curve as a contour line defined by  $\{x, y\}$  such that  $f(x, y) = k$  where  $k$  is a real-valued constant. Note that in the following examples, *MAPLE* code is preceded by the ">" symbol. One may include comments by omitting this symbol.

---

<sup>1</sup> Associate Professor and Chair, Department of Economics, Rubel School of Business, Bellarmine University, 2001 Newburg Road, Louisville, KY, USA 40205, Tel. 502-452-8487, Fax. 502-452-8013, [fraymond@bellarmine.edu](mailto:fraymond@bellarmine.edu)

<sup>2</sup> Professor of Mathematics, Bellarmine University, [araymond@bellarmine.edu](mailto:araymond@bellarmine.edu)

<sup>3</sup> Professor of Economics, Bellarmine University, [mmccrickard@bellarmine.edu](mailto:mmccrickard@bellarmine.edu)

Example 1:

Graph of Cobb-Douglas Utility: The Indifference Map

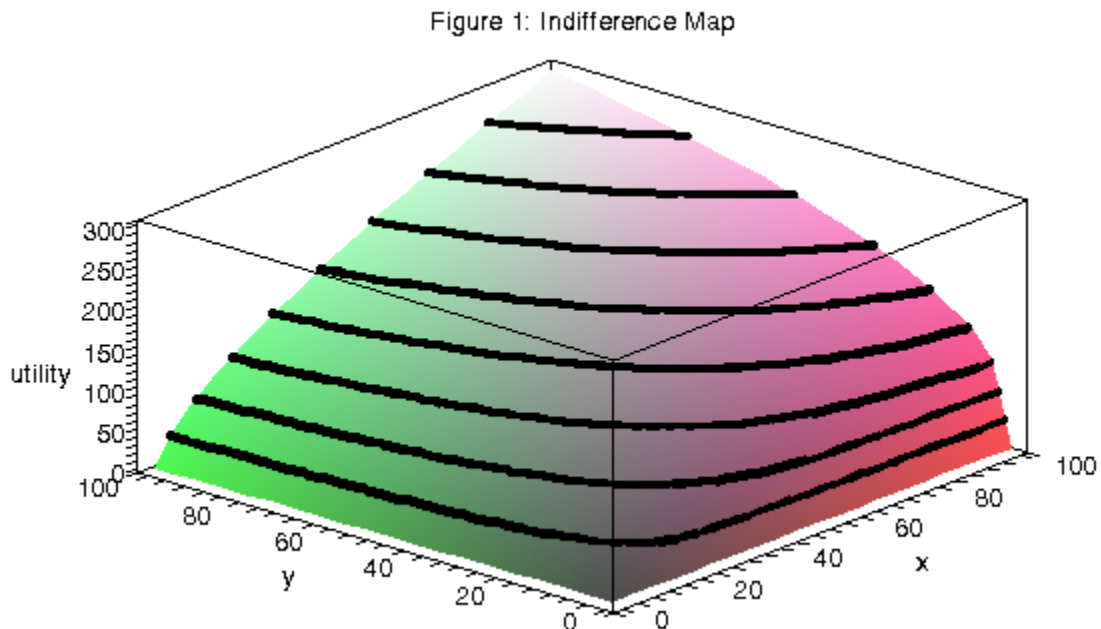
```
> restart;
```

```
> u:=3*x^(2/3)*y^(1/3);
```

$$u := 3 x^{(2/3)} y^{(1/3)}$$

```
>
```

```
plot3d(u,x=0..100,y=0..100,labels=["x","y","utility"],title="Figure 1: Indifference Map",axes=boxed,thickness=4);
```



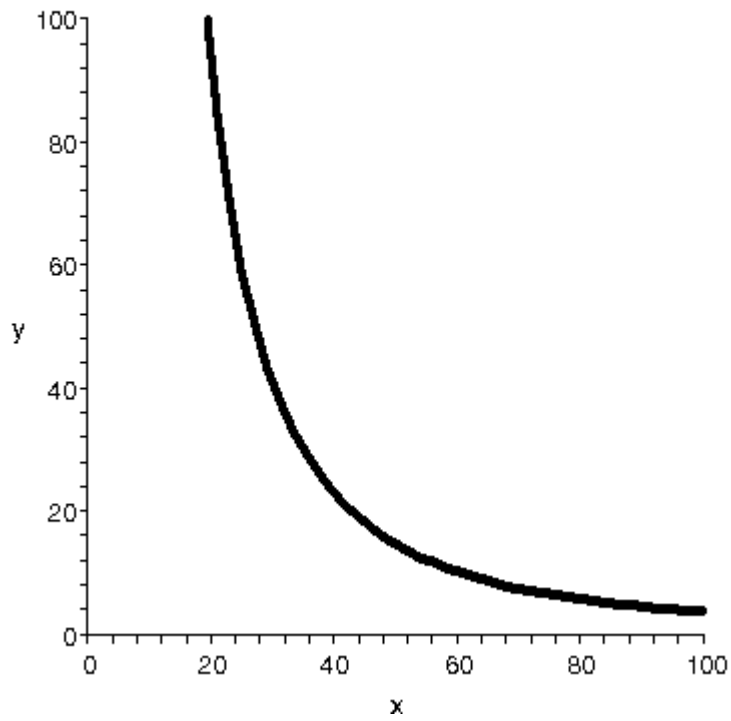
Graphing Indifference Curves

```
> y1:=solve(u=100,y);
```

$$y1 := \frac{1000000}{27 x^2}$$

```
> plot(y1,x=0..100,y=0..100,labels=["x","y"],title="Figure 2: Indifference Curve for 100 utils",color=black,thickness=4);
```

Figure 2: Indifference Curve for 100 utils



The next example illustrates a solution to the consumer utility maximization problem using Lagrange multipliers. The *MAPLE* syntax helps one to clearly distinguish the felicity (function to be optimized) from the budget constraint.

Example 2:

Example of Consumer Optimization with Quasi-linear Preferences

> restart;

Type in utility function.

> u:=y+x^(1/2);

$$u := y + \sqrt{x}$$

Type in the price of good x.

> p[x]:=5;

$$p_x := 5$$

Type in the price of good y.

> p[y]:=7;

$$p_y := 7$$

Type in the amount in the budget, M.

> M:=20;

$$M := 20$$

> Mo:=p[x]\*x+p[y]\*y-M;

$$Mo := 5x + 7y - 20$$

Set up the Lagrangian.

> L:=u-lambda\*(Mo);

$$L := y + \sqrt{x} - \lambda(5x + 7y - 20)$$

Find the first order condition of the Lagrangian with respect to x.

> Diff(L,x);

$$\frac{\partial}{\partial x}(y + \sqrt{x} - \lambda(5x + 7y - 20))$$

> Lx:=value(%);

$$Lx := \frac{1}{2\sqrt{x}} - 5\lambda$$

Find the first order condition of the Lagrangian with respect to y.

> Diff(L,y);

$$\frac{\partial}{\partial y}(y + \sqrt{x} - \lambda(5x + 7y - 20))$$

> Ly:=value(%);

$$Ly := 1 - 7\lambda$$

The following solution (soln) is the optimal consumer bundle.

> soln:=solve({Lx=0,Ly=0,Mo=0},{x,y,lambda});

$$\text{soln} := \left\{ y = \frac{351}{140}, \lambda = \frac{1}{7}, x = \frac{49}{100} \right\}$$

> umx:=eval(u,soln);

$$umx := \frac{351}{140} + \frac{1}{100} \sqrt{49} \sqrt{100}$$

The following value is the maximum utility level, indicative of successful consumer optimization.

```
> umax:=evalf(umx);
```

$$umax := 3.207142857$$

Next we specify the budget constraint by defining the budget constraint as a function of y in terms of x and income.

```
> xint:=M/p[x];
```

$$xint := 4$$

```
> yint:=M/p[y];
```

$$yint := \frac{20}{7}$$

```
> BC:=x->yint-(p[x]/p[y])*x;
```

$$BC := x \rightarrow yint - \frac{P_x x}{P_y}$$

```
> indy:=solve(u=umax,y);
```

$$indy := -1. \sqrt{x} + 3.207142857$$

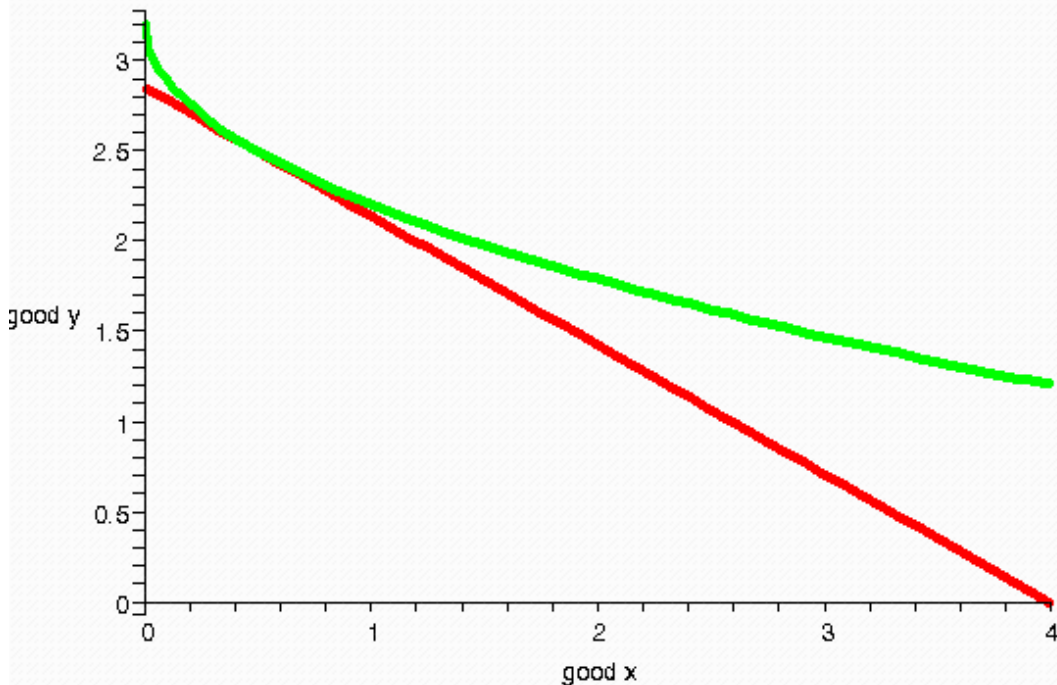
```
> indcv:=x->indy;
```

$$indcv := x \rightarrow indy$$

The graph below illustrates the optimal solution to this consumer problem.

```
> plot([BC(x),indcv(x)],x=0..xint,labels=["good x","good y"],title="Figure 3: Consumer Equilibrium",thickness=3);
```

Figure 3: Consumer Equilibrium



**Examine the utility function and the representative indifference curve depicted here.**

**Can you describe the bias?**

Figure 3 clearly illustrates the bias of the quasi-linear utility function towards good y.

Example three illustrates the similarities and differences between consumer and producer theory. The *MAPLE* syntax indicates that although they illustrate different economic concepts, clearly they are both problems involving constrained maximization. The similarities and differences become more evident when the student is requested to make certain changes to the template provided by the instructor. In order to provide more of a challenge, the instructor may decide to use the consumer's utility maximization problem as the template for the producer's cost minimization problem. Note the slightly more general approach.

Example 3:

This program will determine the Minimum Cost with Cobb-Douglas Production.

```
> restart;
> assume(L,nonnegative);
> assume(K,nonnegative);
> assume(lambda,nonnegative);
>
The Cobb-Douglas Production Function:
> F:=A*L^(delta)*K^(epsilon);
```

$$F := A L^{\delta} K^{\epsilon}$$

```
> "Type in values for A, delta and epsilon.";
      "Type in values for A, delta and epsilon."
```

```
> A:=1;
```

$$A := 1$$

```
> delta:=2/3;
```

$$\delta := \frac{2}{3}$$

```
> epsilon:=1/3;
```

$$\epsilon := \frac{1}{3}$$

Type in the wage.

```
> w:=2;
```

$$w := 2$$

Type in the rental rate.

```
> r:=5;
```

$$r := 5$$

```
>
```

Type in the desired output level.

```
>
```

```
> q:=100;
```

$$q := 100$$

The Cost Function Follows.

```
> C:=r*K+w*L;
```

$$C := 5 K + 2 L$$

```
>
```

The output constraint follows:

Note: This becomes a minimization problem when  $Q_0 \leq 0$ .

```
> Qo:=q-F;
```

$$Q_0 := 100 - L^{(2/3)} K^{(1/3)}$$

```
> Lag:=C+lambda*(Qo);
```

$$\text{Lag} := 5 K_{\sim} + 2 L_{\sim} + \lambda_{\sim} \left( 100 - L_{\sim}^{(2/3)} K_{\sim}^{(1/3)} \right)$$

> Diff(Lag,L);

$$\frac{\partial}{\partial L_{\sim}} \left( 5 K_{\sim} + 2 L_{\sim} + \lambda_{\sim} \left( 100 - L_{\sim}^{(2/3)} K_{\sim}^{(1/3)} \right) \right)$$

> LL:=value(%);

$$LL := 2 - \frac{2 \lambda_{\sim} K_{\sim}^{(1/3)}}{3 L_{\sim}^{(1/3)}}$$

> Diff(Lag,K);

$$\frac{\partial}{\partial K_{\sim}} \left( 5 K_{\sim} + 2 L_{\sim} + \lambda_{\sim} \left( 100 - L_{\sim}^{(2/3)} K_{\sim}^{(1/3)} \right) \right)$$

> LK:=value(%);

$$LK := 5 - \frac{\lambda_{\sim} L_{\sim}^{(2/3)}}{3 K_{\sim}^{(2/3)}}$$

The following solution is the cost-minimizing bundle.

>  
 > soln:=fsolve({LL=0,LK=0,Qo=0},{L,K,lambda});  
 soln := {K\_{\sim} = 34.19951893, L\_{\sim} = 170.9975947, \lambda\_{\sim} = 5.129927840}

>  
 The following value is the minimum cost.

> minc:=eval(C,soln);  
 minc := 512.9927840

> lint:=minc/w;  
 lint := 256.4963920

> kint:=minc/r;  
 kint := 102.5985568

> TC:=L->kint-(w/r)\*L;  
 $TC := L \rightarrow kint - \frac{w L}{r}$

>  
 > isoK:=solve(q=F,K);

$$isoK := \frac{1000000}{L_{\sim}^2}$$

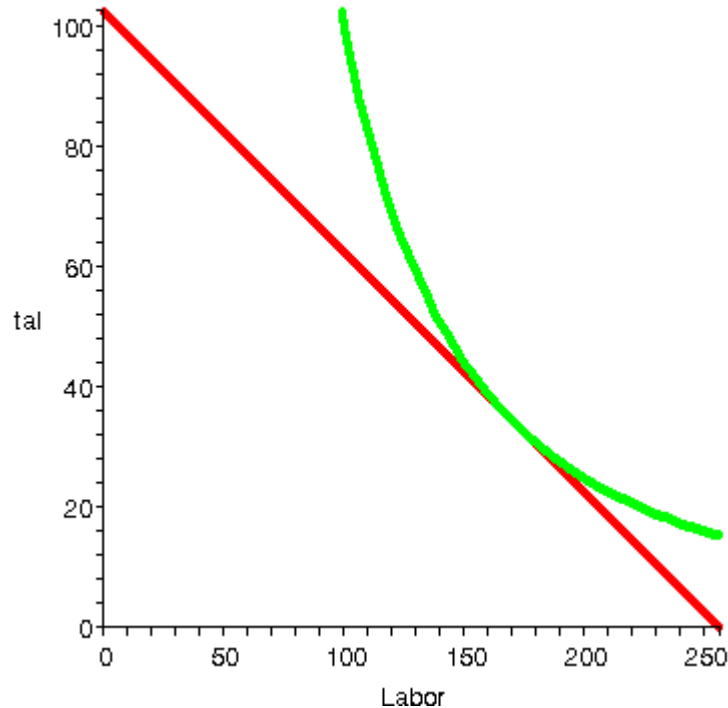
> isoquant:=L->isoK;  
 isoquant := L \rightarrow isoK



The graph below illustrates the optimal solution.

```
>
plot([TC(L),isoquant(L)],L=0..lint,K=0..kint,labels=["Labor",
"Capital"],title="Production Equilibrium",thickness=3);
```

Figure 4: Production Equilibrium



To illustrate that this is indeed the minimum cost, we evaluate the cost of two nearby points of the same isoquant.

```
> L1:=eval(L,soln)-.2;
L1 := 170.7975947
> L2:=eval(L,soln)+.2;
L2 := 171.1975947
> K1:=eval(isoquant(L),L=L1);
K1 := 34.27965949
> K2:=eval(isoquant(L),L=L2);
K2 := 34.11965905
> C1:=eval(w*L1+r*K1);
C1 := 512.9934868
> C2:=eval(w*L2+r*K2);
C2 := 512.9934846
```

Recall that the minimum cost was  $512.9927841 < C1$  and  $C2$ .

In the next example we solve for and demonstrate the Hicksian substitution and income effects.

Example 4:

### Hicks Substitution and Income Effects

```
> assume(x,nonnegative);  
> assume(y,nonnegative);  
> assume(lambda,nonnegative);
```

Type in values for A, delta and epsilon.

```
> A:=1;
```

$$A := 1$$

```
> delta:=1/2;
```

$$\delta := \frac{1}{2}$$

```
> epsilon:=1/2;
```

$$\epsilon := \frac{1}{2}$$

The Cobb-Douglas Utility Function follows.

```
> u:=A*x^(delta)*y^(epsilon);
```

$$u := \sqrt{x} \sqrt{y}$$

Type in the price of good x. We will investigate the effect of a price increase in good x.

```
> p1[x]:=2;
```

$$p1_x := 2$$

```
> p2[x]:=3;
```

$$p2_x := 3$$

Type in the price of good y.

```
> p[y]:=50/8;
```

$$p_y := \frac{25}{4}$$

Type in the budget amount.

```
> M:=100;
```

$$M := 100$$

Write the budget constraints, before and after the increase in the price of x.

```
> M1:=M-(p1[x]*x+p[y]*y);
```

$$M1 := 100 - 2x - \frac{25}{4}y$$

```
> M2:=M-(p2[x]*x+p[y]*y);
```

$$M2 := 100 - 3x - \frac{25}{4}y$$

Determine the first Lagrangian equation.

```
> L1:=u+lambda[1]*(M1);
```

$$L1 := \sqrt{x} \sqrt{y} + \lambda_1 \left( 100 - 2x - \frac{25}{4}y \right)$$

> Diff(L1,x);

$$\frac{\partial}{\partial x} \left( \sqrt{x} \sqrt{y} + \lambda_1 \left( 100 - 2x - \frac{25}{4}y \right) \right)$$

> L1x:=value(%);

$$L1x := \frac{\sqrt{y}}{2\sqrt{x}} - 2\lambda_1$$

> Diff(L1,y);

$$\frac{\partial}{\partial y} \left( \sqrt{x} \sqrt{y} + \lambda_1 \left( 100 - 2x - \frac{25}{4}y \right) \right)$$

> L1y:=value(%);

$$L1y := \frac{\sqrt{x}}{2\sqrt{y}} - \frac{25}{4}\lambda_1$$

Next, determine the Lagrangian equation after the price of x rises.

> L2:=u+lambd[2]\*(M2);

$$L2 := \sqrt{x} \sqrt{y} + \lambda_2 \left( 100 - 3x - \frac{25}{4}y \right)$$

> Diff(L2,x);

$$\frac{\partial}{\partial x} \left( \sqrt{x} \sqrt{y} + \lambda_2 \left( 100 - 3x - \frac{25}{4}y \right) \right)$$

> L2x:=value(%);

$$L2x := \frac{\sqrt{y}}{2\sqrt{x}} - 3\lambda_2$$

> Diff(L2,y);

$$\frac{\partial}{\partial y} \left( \sqrt{x} \sqrt{y} + \lambda_2 \left( 100 - 3x - \frac{25}{4}y \right) \right)$$

> L2y:=value(%);

$$L2y := \frac{\sqrt{x}}{2\sqrt{y}} - \frac{25}{4}\lambda_2$$

The following solutions (soln1 and soln2) are the optimal bundles before and after the rise in the price of x.

```
> soln1:=solve({L1x=0,L1y=0,M1=0},{x,y,lambda[1]});
soln1 := { λ~1 = RootOf(-1 + 50 _Z^2, 0.1414213562), x~ = 25, y~ = 8 }
```

```
> soln2:=solve({L2x=0,L2y=0,M2=0},{x,y,lambda[2]});
soln2 := { λ~2 = RootOf(-1 + 75 _Z^2, 0.1154700538), y~ = 8, x~ = 50/3 }
```

```
> umx1:=eval(u,soln1);
```

$$umx1 := \sqrt{25} \sqrt{8}$$

```
> umx2:=eval(u,soln2);
```

$$umx2 := \frac{1}{3} \sqrt{50} \sqrt{3} \sqrt{8}$$

The values for umax1 and umax2 denote the optimal levels of utility before and after the price increase.

```
> umax1:=evalf(umx1);
```

$$umax1 := 14.14213562$$

```
> umax2:=evalf(umx2);
```

$$umax2 := 11.54700539$$

Here are the x and y intercepts for the two budget constraints.

```
> x1int:=round(M/p1[x]);
```

$$x1int := 50$$

```
> x2int:=round(M/p2[x]);
```

$$x2int := 33$$

```
> yint:=M/p[y];
```

$$yint := 16$$

Arrange in functional form for the purpose of graphing the budget constraints.

```
> BC1:=x->yint-(p1[x]/p[y])*x;
```

$$BC1 := x \rightarrow yint - \frac{p1_x x}{p_y}$$

```
> BC2:=x->yint-(p2[x]/p[y])*x;
```

$$BC2 := x \rightarrow yint - \frac{p2_x x}{p_y}$$

Solve and graph the two indifference curves indicating utility before and after the price increase.

```
> indy1:=solve(u=umax1,y);
```

$$indy1 := \frac{199.9999999}{x~}$$

```
> indy2:=solve(u=umax2,y);
```

$$\text{indy2} := \frac{133.3333335}{x}$$

```
> indcv1:=x->indy1;
```

$$\text{indcv1} := x \rightarrow \text{indy1}$$

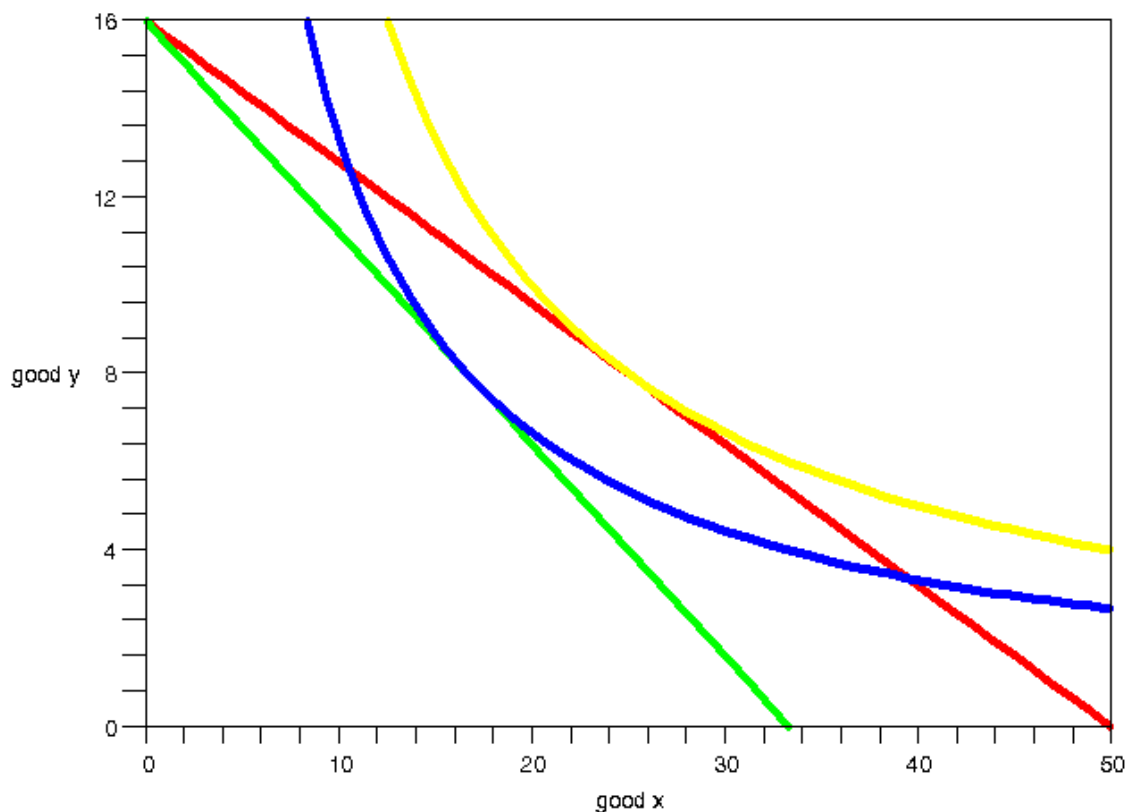
```
> indcv2:=x->indy2;
```

$$\text{indcv2} := x \rightarrow \text{indy2}$$

The graph below illustrates the optimal solutions before (red and yellow tangency) and after (green and blue tangency) the price of x increases.

```
> plot([BC1(x),BC2(x),indcv1(x),indcv2(x)],x=0..xlint,y=0..yint,labels=["good x","good y"],title="Figure 5: Consumer Equilibrium",thickness=4,axes=boxed);
```

Figure 5: Consumer Equilibrium



To identify the Hicks substitution and income effects for x, look at the choice the consumer would make if allowed to maintain original utility level at the new price ratio.

```
> pr:=p2[x]/p[y];
```

$$pr := \frac{12}{25}$$

```
> u1:=umax1-u;
```

$$u1 := 14.14213562 - \sqrt{x} - \sqrt{y}$$

```
> u1y:=solve(u1=0,y);
```

$$u1y := \frac{199.9999999}{x}$$

Find the marginal rate of substitution.

```
> Diff(u1y,x);
```

$$\frac{d}{dx} \left( \frac{199.9999999}{x} \right)$$

```
> u1slope:=-value(%);
```

$$u1slope := \frac{199.9999999}{x^2}$$

Use the fact that at the point of tangency, the price ratio equals the marginal rate of substitution in order to find the x and y values associated with the Hicksian solution.

```
> Hicks:=solve(pr=u1slope,x);
```

$$Hicks := 20.41241452, -20.41241452$$

```
> Hicksx:=max(Hicks);
```

$$Hicksx := 20.41241452$$

```
> Hicksy:=eval(u1y,x=Hicksx);
```

$$Hicksy := 9.797958967$$

Numerically solve for the Hicks substitution and income effects.

```
> x1:=solve(soln1[3]);
```

$$x1 := 8$$

```
> SubEffect:=x1-Hicksx;
```

$$SubEffect := -12.41241452$$

```
> x2:=solve(soln2[3]);
```

$$x2 := \frac{50}{3}$$

```
> IncEffect:=Hicksx-x2;
```

$$IncEffect := 3.74574785$$

Next, rearrange format for graphing the Hicksian budget constraint.

```
> Mg:=p2[x]*Hicksx+p[y]*Hicksy;
```

$$Mg := 122.4744871$$

```
> BC3:=x->(Mg/p[y])-(p2[x]/p[y])*x;
```

$$BC3 := x \rightarrow \frac{Mg}{p_y} - \frac{p2_x x}{p_y}$$

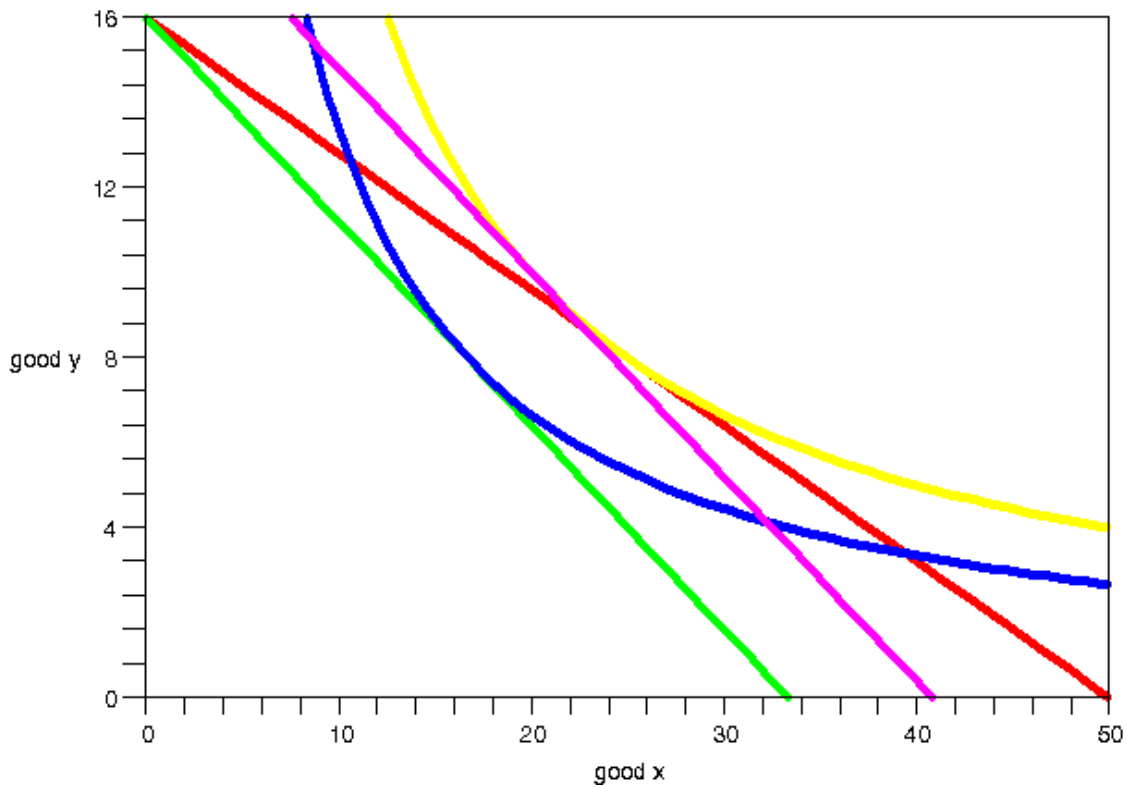
The graph below indicates the Hicks substitution and income effects

```
>
```

```
plot([BC1(x),BC2(x),indcv1(x),indcv2(x),BC3(x)],x=0..xlint,  
y=0..yint,labels=["good x","good y"],title="Figure 6: Hicks
```

Substitution and Income Effects for Good  
x", thickness=4, axes=boxed);

Figure 6: Hicks Substitution and Income Effects for Good x



**Questions:**

**On the graph,**

- 1) identify the original optimal bundle,**
- 2) identify the optimal bundle after the price change,**
- 3) identify the change in good x that describes the substitution effect,**
- 4) identify the change in good x that describes the income effect.**
- 5) Repeat (3) and (4) for good y.**

The next four examples come from intermediate macroeconomic theory. The first example depicts the Keynesian Cross. The second solves the Keynesian IS-LM model. The final two examples illustrate the Solow model of economic growth, including the Golden Rule.

Example 5:

### The Keynesian Cross

> restart;

> assume(Y1>=0,TAX>=0,INV>=0,GOV>=0,IM>=0);

This program allows us to look at nonlinear versions of the model.

>

Government Expenditures and the Marginal Tax Rate

>

> GOV:=1000-0.01\*Y;

$$GOV := 1000 - 0.01 Y$$

> TAX:=0.05\*Y;

$$TAX := 0.05 Y$$

>

The Consumption Function

>

> CONS:=3000+15\*(Y-TAX)^0.5;

$$CONS := 3000 + 14.62019152 Y^{0.5}$$

The Investment Schedule

> INV:=1000+0.1\*Y;

$$INV := 1000 + 0.1 Y$$

>

Exogenous Exports

> EX:=600;

$$EX := 600$$

>

Imports

>

> IM:=400+0.1\*(Y-TAX);

$$IM := 400 + 0.095 Y$$

Define the Keynesian Cross

> AGGEXP:=CONS+GOV+INV+EX-IM;

$$AGGEXP := 5200 + 14.62019152 Y^{0.5} - 0.005 Y$$

>

Equilibrium GDP for this Model

> GDP:=solve({AGGEXP=Y});

$$GDP := \{Y = 6331.699000\}$$

Define Savings

> SAV:=INV+GOV+EX-IM-TAX;

$$SAV := 2200 - 0.055 Y$$

Define Disposable Income

> DISPINC:=CONS+SAV;

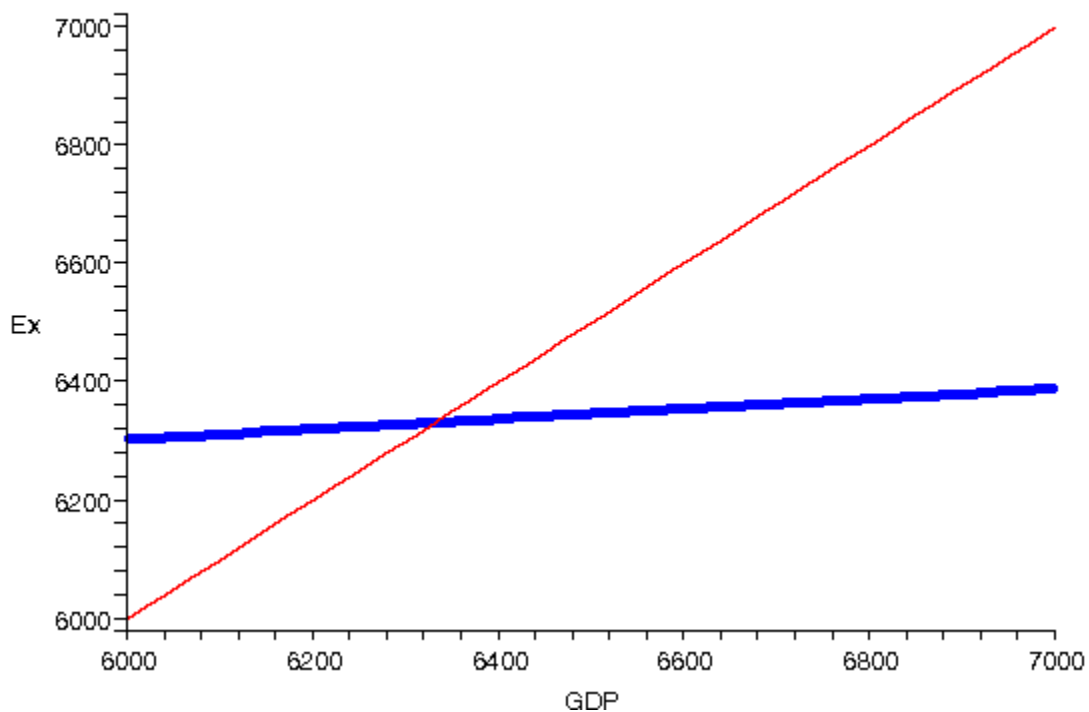
$$DISPINC := 5200 + 14.62019152 Y^{0.5} - 0.055 Y$$



Evaluate these parameters with respect to equilibrium GDP.

```
> eval([Y,SAV,DISPINC],GDP);  
      [6331.699000, 1851.756555, 6015.114050]  
> eval([TAX,INV,GOV,IM,EX],GDP);  
      [316.5849500, 1633.169900, 936.6830100, 1001.511405, 600]  
>  
p1:=plot([AGGEXP],Y=6000..7000,labels=["GDP","Ex"],title="Figure 7, Aggregate Expenditures",thickness=4,color=blue):  
> f:=Y->Y;  
       $f: Y \rightarrow Y$   
> p2:=plot([f(Y)],Y=6000..7000,thickness=2):  
> with(plots):  
Warning, the name changecoords has been redefined  
> display(p1,p2);
```

Figure 7, Aggregate Expenditures



Example 6:

## The Keynesian IS-LM Model

```
> restart;
```

### Government Expenditures and the Marginal Tax Rate

```
> GOV:=10000;
```

$$GOV := 10000$$

```
> TAX:=0.05*Y;
```

$$TAX := 0.05 Y$$

### The Consumption Function

```
> CONS:=8000+.85*(Y-TAX);
```

$$CONS := 8000. + 0.8075 Y$$

### The Investment Schedule

```
> INV:=2000-50*r;
```

$$INV := 2000 - 50 r$$

### Exports

```
> EX:=6000;
```

$$EX := 6000$$

### Imports

```
> IM:=5000+0.05*(Y-TAX);
```

$$IM := 5000. + 0.0475 Y$$

### Determine the Investment-Savings (IS) Function.

```
> AGGEXP:=CONS+GOV+INV+EX-IM;
```

$$AGGEXP := 21000. + 0.7600 Y - 50 r$$

```
> IS:=solve(Y=AGGEXP,r);
```

$$IS := -0.0048000000000 Y + 420.$$

```
> p1:=plot(IS, Y=85000..90000,r=0..15,thickness=5):
```

```
>
```

```
>
```

### Given the Money Supply.

```
> MS:=8500;
```

$$MS := 8500$$

### Given Money Demand.

```
> MD:=40+0.1*Y-30*r;
```

$$MD := 40 + 0.1 Y - 30 r$$

### Solve for Liquidity-Money (LM) Curve.

```
> LM:=solve(MS=MD,r);
```

$$LM := -282. + 0.0033333333333 Y$$

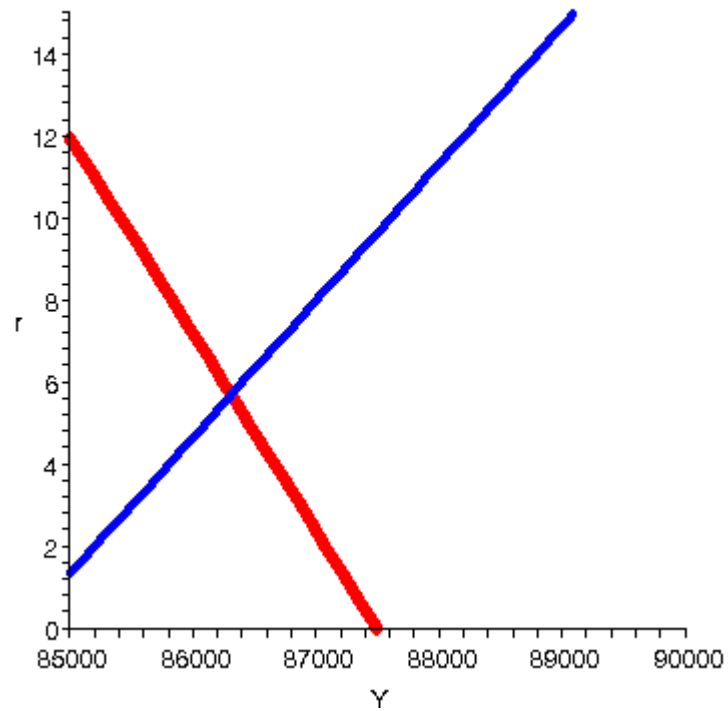
```
>
```

```
> p2:=plot(LM, Y=85000..90000,r=0..15,title="Figure 8: The  
Keynesian IS-LM Curves",thickness=4,color=blue):
```

```
> with(plots):
```

```
> display(p1,p2,labels=["Y","r"]);
```

Figure 8: The Keynesian IS-LM Curves



**Solve for the Equilibrium GDP and Interest Rate.**

**Equilibrium GDP:**

```
> eY:=solve(IS=LM,{Y});  
eY := {Y = 86311.47541}
```

**Equilibrium Interest Rate:**

```
> er:=eval(LM,eY);  
er := 5.7049180
```

Example 7:

## The Solow Growth Model

Though not essential for this application, the purpose of the inactive code (in black) is to provide related information and to more clearly reveal how the model is constructed.

```
> restart;
```

1) Define a CRS Production Function.

```
F:=(K,L)->A*K^.5*L^.5;
```

```
Diff(F(K,L),L);
```

```
MPL:=value(%);
```

```
Diff(F(K,L),K);
```

```
MPK:=value(%);
```

```
> A:=3;
```

```
A := 3
```

```
k:=K/L;
```

```
y:=Y/L;
```

Redefine Production in terms of Relative Capital.

```
> f:=k->A*k^.5;
```

```
f := k -> A k0.5
```

2) Incorporate Investment, Consumption and Savings.

```
Y:=C+S;
```

```
C:=(1-sigma)*Y;
```

```
c:=C/L;
```

```
s:=S/L;
```

```
i:=I/L;
```

Leakages = Injections in Equilibrium.

```
s:=i;
```

Let sigma represent the marginal propensity to save. Then, 1-sigma is the marginal propensity to consume.

```
c:=(1-sigma)*y;
```

```
c:=y-s;
```

```
i=sigma*y;
```

```
> sigma:=.15;
```

```
σ := 0.15
```

3) The Capital Constraint

```
ChangeK:=I-delta*k;
```

```
> delta:=.10;
```

```
δ := 0.10
```

```
> changek:= sigma*f(k)-delta*k;
```

```
changek := 0.45 k0.5 - 0.10 k
```

changek is the change in effective (K/L) capital.

#### 4) Consumption

```
> cc:=f(k)-changek-delta*k;
```

$$cc := 2.55 k^{0.5}$$

cc is consumption.

```
> p1:=plot(f(k),k=1..1000,title="Figure 9: Production and  
Consumption",color=blue,thickness=3):
```

```
> p2:=plot(delta*k,k=1..1000, thickness=4):
```

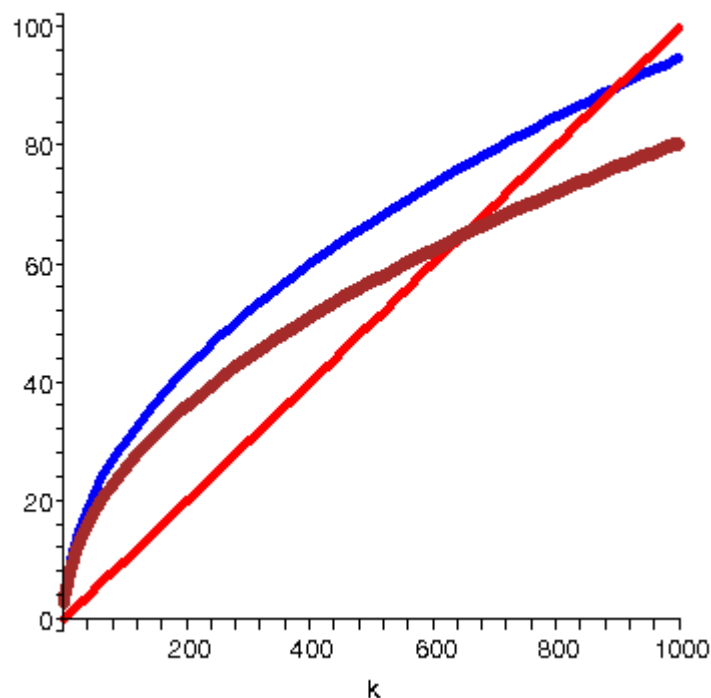
```
> p3:=plot(cc,k=1..1000, color=brown,thickness=5):
```

Production,  $f(k)$ , is in blue. Consumption is in brown. Depreciation is in red.

```
> with(plots):
```

```
> display(p1,p2,p3);
```

Figure 9: Production and Consumption



The final example is an extension of example 7. It depicts the Golden Rule for capital accumulation. We derive the steady state level of capital necessary to maximize per capita consumption over time.

Example 8:

**The Golden Rule: Determining Optimal Steady State Consumption from the Solow Growth Model**

```
> restart;
> A:=4;
                                A := 4
> f:=k->A*k^.5;
                                f := k → A k0.5
>
> delta:=.10;
                                δ := 0.10
> Diff(f(k),k);
                                 $\frac{d}{dk}(4 k^{0.5})$ 
> MPk:=value(%);
                                MPk :=  $\frac{2.0}{k^{0.5}}$ 
> grule:=solve(MPk=delta,k);
                                grule := 400.
```

The Golden Rule: grule is the optimal steady state level of capital associated with the highest possible sustainable level of consumption. It is given by the intersection of the savings (green) and depreciation (red) curves below.

```
> sigma:=delta*grule/f(grule);
                                σ := 0.50000000000
> Sigma is the optimal savings rate that guarantees a
maximum steady state stream of consumption.
> p1:=plot(f(k),k=1..1000,title="Figure 10: Golden Rule
Production",color=blue,thickness=3):
> p2:=plot(delta*k,k=1..1000,thickness=4):
> p3:=plot(sigma*f(k),k=1..1000,color=green,thickness=5):
> with(plots):

> Production, f(k), is in blue. Depreciation is red.
Savings is green.
> display(p1,p2,p3);
```

Figure 10: Golden Rule Production

